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OPTIMAL DESIGN OF SANDWICH BEAMS

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Abstract— Effective cost reduction of manufacture structures can be achieved by applying mathematical methods of optimal design. This article presents the determination of optimal geometrical sizes of three-layer sandwich beams with profile. It reviews the formulation and design constraints of the objective function. The objective function includes the manufacturing costs, while the design constrains refer to the maximal normal and shear stresses, the maximal deflection, damping of vibration and geometrical sizes. It examines the changes of the optimal geometrical sizes and that of the minimum objective function as a function of loading.

Keywords— Effective cost reduction, objective function, optimal geometrical sizes, sandwich beams.

I. INTRODUCTION

THE sandwich structures (beams, panels and shells) are widely using of different area of industry and economy in this days. Their advantage is that in different shape relative fast could make and assemble. Their small weight they are very good at the vibration dampers and heat isolation. The outside skins (shells) could make with profile plates or panels so their stiffness are significantly increase.

II. THE OBJECTIVE FUNCTION AND DESIGN CONSRAINS

In the mechanical engineering practice the optimization process is divided into process and structural optimization. The structural optimization we can divide structural topology, shape, sizing, and material optimization. The optimization of structural topology is happened by truss-girder in the first place when have to define the elements (rods) of girder a good place in the space. At the shape optimization it is our aim that the shape of structure is followed the expected shape of the design constrains (e.g. the arms of tree are the most thick where the bending moment and shear forces are the largest). Topology optimization of trusses in the form of grid-like is a classical subject in structural design. The study of fundamental properties of optimal grid like continua was pioneered by Michell, 1904, but this interesting field has only much later developed into what is now the well-established lay-out theory for frames and flexural systems [2].

The optimization of geometrical sizes are the most frequently tasks when have to decide the optimal sizes of cross-section. By the optimization of material consumption are the same as the optimization of structural topology, but in this case have to look for the optimal material setting in order. The optimization of material we use often to the designing of composite material. The process optimization is used by the technological process to decide the optimal parameters of process [1]. We show the optimal design of the simply supported sandwich beam with uniformly distributed load, *Fig. 1*. The length of beam is *l*. We use a sandwich beam which consists of two aluminium/steel rectangular hollow sections (RHS) and a layer poliurethane foam (h_2) glued between them.

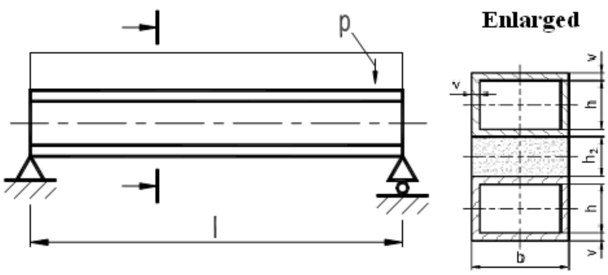

Fig. 1. The construction of model and the cross-section of sandwich beams.

A. Formulation of objective function

The objective function consist of the following elements: cost of the cutting of the hollow section (K_d), cost of the cleaning of interface of core (K_c), the costs of foam and its working time cost (K_f) and the material costs of hollow section (K_m). So the costs are [2], [5], [6], [7].

$$\mathbf{K} = \mathbf{K_d} + \mathbf{K_c} + \mathbf{K_f} + \mathbf{K_m}. \quad (1)$$

The cost of cutting of the hollow sections

$$\mathbf{K_d} = \mathbf{n_d} \cdot \mathbf{k_d}, \quad (2)$$

where n_d number of cuttings, k_d specific cost of cutting (HUF/cutting). The cleaning cost of interface of core is

$$\mathbf{K_c = k_{fk} \cdot A_t,} \tag{3}$$

where k_{fa} the cleaning costs of the surface by hand (HUF/m^2) and A_t is the cleaned surface (m^2).
The costs of core with using the equipment foam manufacturer we can calculate

$$\mathbf{K_f = \left(t_e + \frac{blh_2}{V_h} + t_{ki}\right)\left(k_m + k_{afoam}\right) + h_2 lbk_{foam},} \tag{4}$$

where t_e preparation time (min), h_2 thickness of polyurethane foam [mm], V_h volume flow of polyurethane foam (m^3/s), t_{ki} operating time of machine (min), b the width of hollow section [mm], k_m specific labor cost (HUF/min), k_{afoam} specific amortization cost of machine (HUF/min), k_{foam} specific material cost of foam (HUF/m^3).
The material cost of hollow sections

$$\mathbf{K_a = k_{fa} l,} \tag{5}$$

where k_{fa} the specific material cost per unit length (HUF/m).

B. Formulation of constraints

1. The constraint for the maximal deflection of beam

The maximum value of deflection [3]

$$\mathbf{w_{max} = \frac{5pl^4}{384EI} + \frac{pl^2}{8AG}\left(1 - \frac{I_f}{I}\right)^2 \psi_1,} \tag{6}$$

where

$$\mathbf{I = I_f + 2A\left(\frac{d}{2}\right)^2,\ I_f = 2I_1,\ I_1 = \frac{vh^3}{6} + \frac{bv^3}{6} + \frac{bvh^2}{2},} \tag{7}$$

where A the cross section area of closed section, v is the thickness and h is the height of hollow section, I_1 the second moment of area for closed section, E modulus of elasticity of facings, G shear modulus of core, p uniformly distributed load (N/mm),

$$\mathbf{d = \frac{h_2}{2} + 2v + h, \quad A = \frac{b\left(h_2 + 2v + h\right)^2}{h_2},} \tag{8}$$

where

$$\mathbf{\psi_1 = 1 + \frac{2\beta_2}{\theta}\left(1 - \cos\theta\right), \quad \theta = \frac{1}{2}\sqrt{\frac{AG}{EI_f\left(1 - \frac{I_f}{I}\right)}},} \tag{9}$$

$$\mathbf{\beta_2 = \frac{1/\theta + th\varphi}{\cos\theta + sh\theta \cdot th\varphi},} \tag{10}$$

where $\varphi = 0$.
The deflection constraint is given by

$$\mathbf{w_{max} \le w_{adm},} \tag{11}$$

where $w_{adm} = l/300$ is the admissible deflection of beam.

2. The constraint for the maximal shear stress in the core

The maximal shear stress in the core can be described

$$\mathbf{\tau_{max} = \frac{pl}{2bd}\left(1 - \frac{I_f}{I}\right)\psi_3, \quad \psi_3 = \frac{1}{\beta_2\theta},} \tag{12}$$

$$\mathbf{\tau_{max} \le \tau_{adm},} \tag{13}$$

where τ_{adm} is the admissible shear stress [3].

3. The constraint for the maximal normal stress in the facings

The maximal normal stress in the facings can be described

$$\mathbf{\sigma_{max} = \frac{pl^2}{8}\left[\left(\frac{h_2}{2} + 2v + h\right)\frac{\psi_2}{I} + \frac{h + 2 \cdot v}{2 \cdot I_f}\left(1 - \psi_2\right)\right],} \tag{14}$$

$$\mathbf{\psi_2 = 1 - \frac{2}{\theta^2}\left(1 - \beta_2\theta\right),} \tag{15}$$

$$\mathbf{\sigma_{max} \le \sigma_{adm},} \tag{16}$$

where σ_{adm} is the admissible normal stress.

4. The constraint for the loss factor of the sandwich beam

The loss factor of three-layered sandwich beam can be expressed

$$\mathbf{\eta = \frac{\eta_2 XY}{1 + X\left(2 + Y\right) + X^2\left(1 + Y\right)\left(1 + \eta_2^2\right)},} \tag{17}$$

where

$$\mathbf{Y = \frac{d^2}{2 \cdot B_f} AE, \quad B_f = EI_f, \quad X = g_0 r_k^2,} \tag{18}$$

$$\mathbf{g_0 = \frac{2G_{2d}b}{AEh_2}, \quad r_k = \frac{Cl}{2\pi},} \tag{19}$$

where η_2 is the loss factor of polyurethane foam [-], G_{2d} is the dynamic shear modulus of foam, $C=2$ for simply supported beam (-). The loss factor of sandwich beam is expressed [5]

$$\eta_{min} \le \eta, \tag{20}$$

where η_{min} is the minimal value of loss factor for the sandwich beam.

5. Geometrical constraints

$$h_{2min} \le h_2, \quad h_2 \le h_{2max}. \tag{21}$$

III. Numerical Data and Results

In the next we show the answer of the problem definition and we are studying how change the optimal thickness of foam (h_2) and minima of the objective function (K) in the function of uniformly distributed load (p). We have taken count to the facing aluminium and steel and the length of beam were 1 meter and 2 meter. Given data: n_d = 4; k_d = 80 Ft; k_{fk} = 50 HUF/m^2; t_e = 5 min; t_{ki} = 2 min; k_m = 70 HUF/min; k_{afoam} = 10 HUF/min; k_{foam} = 2600 HUF/m^3; k_{fa} = 819 HUF/m^3; l = 1000 mm, respectively 2000 mm; E = 70 GPa for aluminium, E = 210 GPa for steel ; G = 3,1 MPa (the measurement of G shows the *Fig. 4.*); v = 3 mm; b = 30 mm; τ_{adm} = 0,25 MPa; σ_{adm} = 120 MPa for aluminium, σ_{adm} = 200 MPa for steel; η_2 = 0,22; G_{2d} = 0,69 MPa; C = 2, h_{2min} = 20 mm, h_{2max} = 120 mm, k_{fa}=819 (HUF) in case of aluminium 30x30x2 mm, 1291 (HUF) in case of aluminium 30x30x3 mm, 430 (HUF) in case of steel 30x30x2 mm.

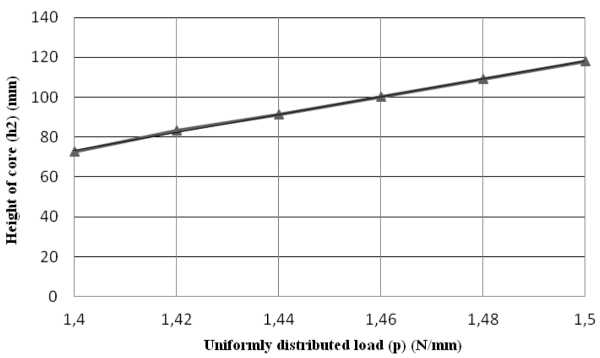

Fig. 2. Changing of the thickness of foam in the function of uniformly distributed load. The material is aluminium and l is 1000 mm.

The minima of the objective function (K) has grown in the examined domain only in small degree. The counted results of foam thickness show the *Fig.2., Fig.3., Fig.6.* and *Fig. 7.* and the counted costs show the *Fig. 8.* and *Fig. 9.* The bending of beam shows the *Fig. 5.*

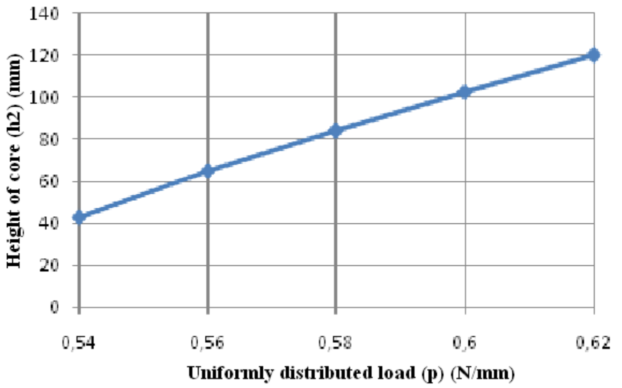

Fig. 3. Changing of the thickness of foam in the function of uniformly distributed load. The material is aluminium and l is 2000 mm.

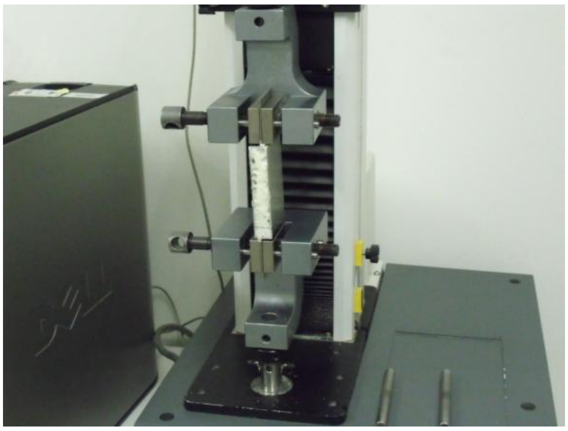

Fig. 4. Measuring of the G shear modulus of core

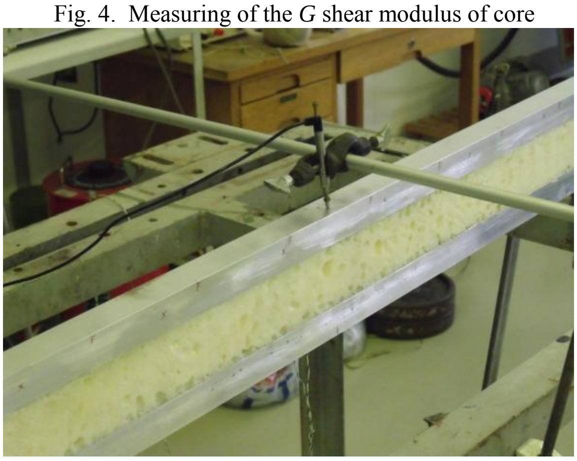

Fig. 5. Measuring of the bending of beam

There is contradiction between stiffness and vibration damping of load-carrying structures. Very stiff structures have low damping capacity, and a high damping ratio can be achieved by permission of some displacements, which decrease the stiffness. A simple welded steel or aluminium-alloy beam has a very small vibration damping capacity, thus, in several applications a sandwich beam may be used, which consist of two aluminium rectangular hollow section and a layer of high damping material (e.g. rubber) glued between them [2].

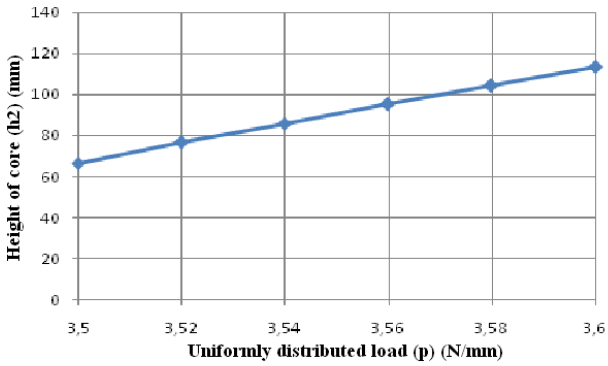

Fig. 6. Changing of the thickness of foam in the function of uniformly distributed load. The material is steel and *l* is 1000 mm.

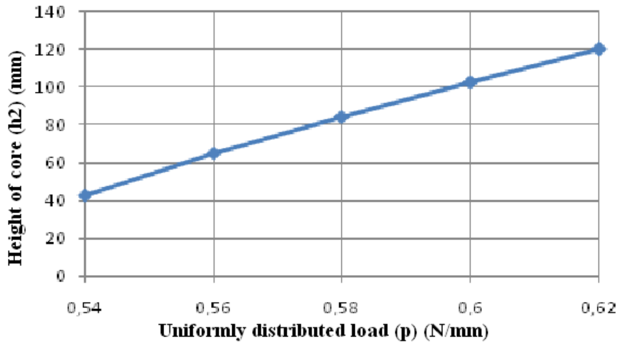

Fig. 7. Changing of the thickness of foam in the function of uniformly distributed load. The material is steel and *l* is 2000 mm.

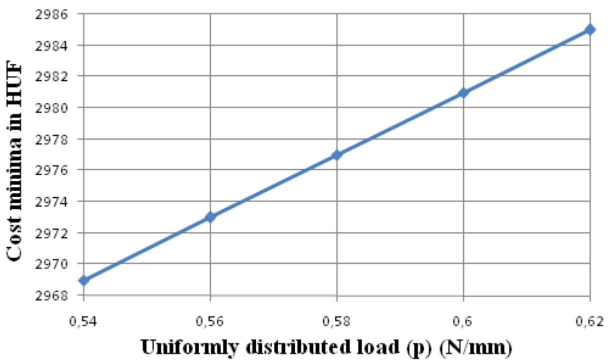

Fig. 8. Changing of the costs in the function of uniformly distributed load. The material is steel and *l* is 2000 mm.

In the optimization procedure the unknown dimension was the height of foam and we changed the sizes of rectangular hollow sections. The objective function was the minima of costs. The measurements show that, to reach a high structural stiffness and a high damping capacity. Using a god damping core, the loss factor of the beam can be large. In this sense the sandwich is better solution, than the simple aluminium beam.

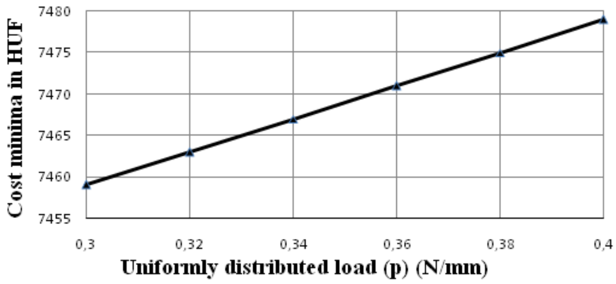

Fig. 9. Changing of the costs in the function of uniformly distributed load. The material is aluminium and *l* is 2000 mm.

The simple aluminium beams are the most expensive ones, since their loss factor is very small and the dynamic force at resonance is very large.

IV. Conclusion

In this work we showed briefly the design optimization of the three-layer sandwich beam with profile in case of mechanical constrains. We determined the optimal thickness of foam core and the changing of minima of cost function in the gear of loads.

Acknowledgment

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